

Dependence Structure of Bivariate Order Statistics And Its Applications

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Abstract

We study the dependence structure of bivariate order statistics from bivariate distributions, and prove that if the underlying bivariate distribution H is positive quadrant dependent (PQD) then so is each pair of bivariate order statistics. As an application, we show that if H is PQD, the bivariate distribution $K_+^{(n)}$, recently proposed by Bairamov and Bayramoglu (2012), is greater than or equal to Baker's (2008) distribution $H_+^{(n)}$, and hence $K_+^{(n)}$ attains a correlation higher than that of $H_+^{(n)}$. We give two explicit forms of the intractable $K_+^{(n)}$ and prove that for all $n \geq 2$, $K_+^{(n)}$ is PQD regardless of H . We also show that if H is PQD, $K_+^{(n)}$ converges weakly to the Fréchet--Hoeffding upper bound as n tends to infinity.